

WHY THE ORDER OF THE FIGURES OF THE HYPOTHETICAL SYLLOGISMS WAS CHANGED

In chapter 6 of Alcinous' *Handbook of Platonism* we find a discussion of categorical, hypothetical, and mixed syllogisms. Alcinous distinguishes three figures of the hypothetical syllogism, and illustrates each figure with a syllogism based on an argument from Plato. Here he remarks in passing that most people called the second hypothetical figure the third and that some called the third figure the second.¹ We may assume that those who called the third figure the second and those who called the second the third were the same. In a parallel passage, Alexander of Aphrodisias advocates the same ordering of figures of hypothetical syllogisms as Alcinous, and reports that Theophrastus, in the first book of his *Analytics*, had the second and third figure in reverse order.² Combining these passages, we can infer that at the turn of the second century A.D. there existed two different views on the ordering of the figures of the hypothetical syllogisms, of which one goes back to Theophrastus, whereas the other presumably was the result of a later change. This curious fact has so far not received a satisfactory explanation.³ In the following pages I seek to show what prompted this reversal of the order of figures. To this end, I first take a look at a

¹ 'And in accordance with the second hypothetical figure (which most people call the third), in which the common term follows the extremes in either premise, he argues thus. . . . And in accordance with the third figure (which some take to be the second), in which the common term precedes the extreme terms, in the *Phaedo* he argues in effect thus . . .' (Alc. *Didasc.* 159.14–15 and 20 Whittaker, *Κατὰ δὲ τὸ δεύτερον ὑποθετικὸν σχῆμα, ὃ οἱ πλείστοι τρίτον φασί, καθ' ὃ ὁ κοινὸς ὅρος ἀμφοτέροις τοῖς ἄκροις ἔπεται, οὕτως ἐρωτᾷ. . . . Καὶ μὴν καὶ κατὰ τὸ τρίτον σχῆμα, πρὸς τινων δὲ δεύτερον, καθ' ὃ ὁ κοινὸς ὅρος ἀμφοτέρων ἡγείται, ἐν τῷ Φαίδωνι οὕτως ἐρωτᾷ δυνάμει. . . .*)

² 'In the first book of his *Analytics*, Theophrastus states that the second figure of the wholly hypothetical <syllogisms> is the one in which the premises begin with the same <term>, and end in different ones, and the third figure is the one in which they begin with different ones, and end in the same. But we set them out the other way around.' (Alex. *An.Pr.* 328.2–5, *Θεόφραστος μέντοι ἐν τῷ προτέρῳ τῶν Ἀναλυτικῶν δεύτερον σχῆμα λέγει ἐν τοῖς δι' ὧν ὑποθετικοῖς εἶναι, ἐν ᾧ ἀρχόμεναι ἀπὸ τοῦ αὐτοῦ αἱ προτάσεις λήγουσιν εἰς ἕτερα, τρίτον δέ, ἐν ᾧ ἀπὸ διαφόρων ἀρχόμεναι λήγουσιν εἰς ταῦτόν. ἀνάπαλιν δ' ἡμεῖς ἐξεθέμεθα.*)

³ J. Barnes, in his 'Terms and sentences: Theophrastus on hypothetical syllogisms', *Proceedings of the British Academy* 69 (1983), 279–326 at 297–8, remarks on this fact, but offers no explanation. He assumes that it reflects a philosophical dispute that may go back to the first century B.C. Peripatetics. Later (at p. 311) he suspects that Theophrastus had no reason for his ordering of the figures, whereas Alexander had an analogy to Aristotle's 'categorical' syllogisms in mind. Similarly, G. Invernizzi, in *Il Didaskalikos di Albino e il medioplatonismo*, vol. 2 (Rome, 1976), 108, n. 21, suggests that Albinus (i.e. Alcinous) switched the figures in order to make them analogous to the categorical syllogisms. J. Dillon, in his commentary on the passage (*Handbook of Platonism* [Oxford, 1993], 82) suggests that the reason for the reversal of the second and third figure was the fact that someone from whom Alcinous draws used Plato's *Parm.* 137d4–9 to show that Plato made use of first figure hypothetical syllogisms, and the immediately following passage *Parm.* 137e–138a1 to show that Plato also employed what was up to then regarded as third figure hypothetical syllogisms, and then, in order to achieve symmetry (i.e. between the passages from the *Parmenides* and the names of the figures) placed what was the third hypothetical figure second. I am not convinced by this account. I. M. Bochenski, in *La Logique de Théophraste* (Fribourg, 1947), 115, followed by A. Graeser, *Die logischen Fragmente des Theophrast* (Berlin, 1973), 99–100, argues that the second figure is logically closer to the first than the third, but as Barnes (p. 311) rightly points out, this is not borne out by our evidence.

passage from Alexander's commentary on Aristotle's *Prior Analytics*, where he compares the (wholly)⁴ hypothetical syllogisms with Aristotelian 'categorical' syllogisms:

A wholly hypothetical <argument>⁵ is as follows: If A, B; if B, C; therefore if A, C. For in these even the conclusion is hypothetical; for instance 'If it is a human being, it is an animal; if it is an animal, it is a substance; therefore, if it is a human being, it is a substance'. Since then in these, too, there must be some middle term, by virtue of which the premises are connected with each other (for otherwise with these too it is impossible to have a deductive pairing <of premises>), this middle term will have three positions in such pairings, too. Now when the middle term is at the end of one of the premises and at the beginning of the other, this will be the first figure: for it will be in this way as it is in the case when the middle term is predicated of one of the extremes, and is subject to the other. For 'ending' and 'following' are analogous to 'being predicated', and 'beginning' to 'being subject'. . . . When the hypothetical premises begin with different <terms> and end in the same, this will be the second figure, being analogous to the second figure in the categorical syllogisms, in which the middle term is predicated of both extremes. . . . for example, 'if human being, animal; if stone, not animal; hence if human being, not stone'. When the premises begin with the same <terms> and end in different ones, this figure will be analogous to the third. For the preceding <term>, which has the place of the subject, is the same in both premises. . . . for example 'if human being, rational; if not human being, non-rational; hence if not rational, non-rational'.⁶

(Alex. *An.Pr.* 326.22–32, 327.2–5, 12–16, 19–20)

ἔστι δὲ δι' ὅλων ὑποθετικός τοιοῦτος· εἰ τὸ A, τὸ B, εἰ τὸ B, τὸ Γ, εἰ ἄρα τὸ A, τὸ Γ· τούτων γὰρ καὶ τὸ συμπέρασμα ὑποθετικόν· οἷον "εἰ ἀνθρώπος ἐστίν, ζῶν ἐστίν, εἰ ζῶν ἐστίν, οὐσία ἐστίν, εἰ ἄρα ἀνθρώπος ἐστίν, οὐσία ἐστίν". ἐπεὶ τοίνυν δεῖ καὶ ἐν τούτοις μέσον τινὰ ὅρον εἶναι, καθ' ὃν συνάπτουσιν αἱ προτάσεις ἀλλήλαις (ἀλλως γὰρ ἀδύνατον καὶ ἐπὶ τούτων συνακτικὴν συζυγίαν γίνεσθαι) οὗτος ὁ μέσος τριχῶς καὶ ἐν ταῖς τοιαύταις συζυγίαις τεθήσεται. ὅταν μὲν γὰρ ἐν ᾗ μὲν τῶν προτάσεων λήγη, ἐν ᾗ δὲ ἄρχηται, τὸ πρῶτον ἔσται σχῆμα· οὕτως γὰρ ἔξει, ὥς καὶ ὅτε τοῦ μὲν τῶν ἄκρων κατηγορεῖτο, τῷ δὲ ὑπέκειτο. ἀνάλογον γὰρ τὸ μὲν λήγειν καὶ ἔπασθαι τῷ κατηγορεῖσθαι, τὸ δὲ ἄρχεσθαι τῷ ὑποκείσθαι. . . . εἰ δὲ ἀπὸ διαφόρων ἀρχόμεναι αἱ ὑποθετικαὶ προτάσεις λήγοιεν εἰς ταῦτό, ἔσται τὸ τοιοῦτον σχῆμα δεύτερον ἀνάλογον ὃν τῷ ἐν τοῖς κατηγοριοῦσι δευτέρῳ, ἐν οἷς ὁ μέσος ὅρος ἀμφοτέρων τῶν ἄκρων κατηγορεῖτο. . . . οἷον "εἰ ἀνθρώπος, ζῶν, εἰ λίθος, οὐ ζῶν, εἰ ἄρα ἀνθρώπος, οὐ λίθος". εἰ δὲ γε ἀπὸ τοῦ αὐτοῦ ἀρχόμεναι αἱ προτάσεις λήγοιεν εἰς ἕτερα, ἔσται ἀνάλογον τοῦτο τὸ σχῆμα τῷ τρίτῳ· τὸ γὰρ ἡγούμενον ὑποκειμένου χώραν ἔχον ἐν ἀμφοτέροις ταῖς προτάσεσι ταῦτόν ἐστιν. . . . οἷον "εἰ ἀνθρώπος, λογικόν, εἰ μὴ ἀνθρώπος, ἀλογον, εἰ μὴ λογικόν ἄρα, ἀλογον".

I assume that A, B, and C stand for terms rather than propositions,⁷ and accordingly, that when Alexander says 'if A, B', this is short for 'if it is A, it is B'. Moreover, I take it that the conditionals 'if (it is) A, (it is) B', were understood as universally

⁴ In line with the main tradition, Alexander calls the arguments which Alcinous names 'hypothetical' 'wholly hypothetical'. They are called 'wholly hypothetical' since all their premises, and their conclusion, are hypothetical propositions; cf. e.g. Philop. *An.Pr.* 243.14–16.

⁵ On pp. 326–8 Alexander throughout speaks of 'the hypothetical' (ὁ ὑποθετικός), and it is unclear whether we should understand 'syllogism' or 'argument', since he repeatedly voices some doubt as to whether the wholly hypotheticals are syllogisms (e.g. *An.Pr.* 326.12–14).

⁶ In fact, the last two examples are given as examples for *valid* arguments in the second and third figure.

⁷ My main reasons for this assumption are the facts that Alexander, like Alcinous, clearly talks about the relation of (middle and extreme) *terms* in the passage (e.g. *An.Pr.* 327.9 τὸ γὰρ Γ μέσος ὢν ὅρος), and that, when he gives examples for the second and third figures, he replaces the letters by terms such as 'human being', 'rational', etc. (*An.Pr.* 327.11–13 and 18–20). On this point I disagree with Barnes (n. 3, 295), who takes Alexander to refer by 'middle term' to the 'middle hypothesis', i.e. to the component proposition shared by the two hypothetical premises. However, what I say in the following can with some (obvious) modifications be upheld even on the assumption that Alexander used A, B, and C to stand for propositions rather than terms.

quantified conditionals which may be paraphrased as ‘if something is A, it is B’, and which thus have exactly two terms (in the Aristotelian sense);⁸ of these I call the first the ‘preceding term’ and the second the ‘following term’.⁹ The three figures are then characterized by premise pairings of the following kinds:

<i>First figure</i>	<i>Second figure</i>	<i>Third figure</i>
If (it is) A, (it is) B	If (it is) A, (it is) B	If (it is) B, (it is) A
If (it is) B, (it is) C	If (it is) C, (it is) B	If (it is) B, (it is) C

We can then see how (wholly) hypothetical syllogisms are analogous to categorical syllogisms. Alexander puts it like this: “ending” and “following” are analogous to “being predicated”, and “beginning” to “being subject” (*An.Pr.* 326.31–2). I interpret this sentence as: ‘Having X as following term in a hypothetical proposition is analogous to having X as predicate term in a categorical premise; and having X as preceding term in a hypothetical proposition is analogous to having X as subject term in a categorical premise.’ On the level of argument schemes, we can accordingly say that for instance in the first figure

If (it is) A, (it is) B		Every A is B
If (it is) B, (it is) C	is analogous to	Every B is C
<hr/> If (it is) A, (it is) C		<hr/> Every A is C

because in both cases there is the underlying sequence of term letters AB, BC, AC. This analogy has in tow a further analogy between the *figures* of the (wholly) hypothetical syllogisms with those of the categorical ones.¹⁰ All first figure hypothetical syllogisms are analogous to first figure categorical ones, and similarly for the second and third figures. This further analogy is based on the position of the middle term in the premise pairings.¹¹ We can illustrate this fact schematically as follows—with the middle term printed in bold:

<i>First figure</i>		<i>Second figure</i>		<i>Third figure</i>
A is B	If (it is) A, (it is) B	A is B	If (it is) A, (it is) B	B is A
B is C	If (it is) B , (it is) C	C is B	If (it is) C, (it is) B	B is C
				If (it is) B , (it is) A
				If (it is) B , (it is) C

(for reasons of simplicity I leave out indicators of quality and quantity).

My contention is that consideration of structural correspondences of the kind just mentioned led to the reversal of the order of the figures. To see how, we have to air two further pieces of information. First, Alexander mentions that Theophrastus called (wholly) hypothetical syllogisms ‘by analogy’.¹² I assume that they were

⁸ In Alcinous, the conditionals are understood in a different way, as involving a designator in the antecedent, and a cross-reference to the designator in the consequent. They can be paraphrased as ‘If it (i.e. this thing *a*) is A, it (*a*) is B’. (See my ‘Wholly hypothetical syllogisms’, *Phronesis* 2000 [forthcoming].) For my present purposes, again, this difference is of no importance.

⁹ Alexander talks about that which precedes (τὸ ἡγούμενον), that which follows (τὸ ἐπόμενον), and about the term which precedes or which follows (e.g. τὸ γὰρ Γ μέσος ὦν ὁρος . . . ἐπόμενος, *An.Pr.* 327.9; ἐν ᾗ ἡγείται ὁ μέσος . . . ἐν ᾗ ἔπεται ὁ μέσος, *An.Pr.* 327.26–7.)

¹⁰ Cf. Alexander’s use of οὕτως . . . ὥς (*An.Pr.* 326.30) and of ἀνάλογον (*An.Pr.* 327.3 and 14).

¹¹ Alcinous calls the middle term the ‘common term’ (*Didasc.* 159.3, 20–1).

¹² οἱ δὲ ὅλων ὑποθετικοί, οὓς Θεόφραστος “κατὰ ἀναλογίαν” λέγει . . . (*An.Pr.* 326.8–9).

so-called, because they were seen as somehow analogous to Aristotle's 'categorical' syllogisms.¹³ At the very least this is the only sort of analogy which Alexander discusses in this context, and he does so shortly after he has given Theophrastus' alternative name for the (wholly) hypothetical syllogisms.

For the second point we have to go back to Aristotle. In his syllogistic, Aristotle formulated his 'categorical' propositions in a way different from that favoured by Alexander, Alcinous, and other later philosophers. For instance, where Alcinous says 'Every A is B' (cf. *Didasc.* 158.15–16),¹⁴ Aristotle would say 'B holds of every A' or 'B is predicated of every A', and *mutatis mutandis* in the cases of all categorical propositions (e.g. Arist. *An.Pr.* 25a17–18, 25b37–9, 26a23). This way of wording was adopted by Theophrastus (e.g. Alex. *An.Pr.* 31.4–10; Philop. *An.Pr.* 48.11–18). For Aristotle and Theophrastus, a syllogism of the first figure, say in *modus Barbara*, would thus have this linguistic form:

A holds of every B
 B holds of every C

 A holds of every C

At Alexander's time, on the other hand, a *modus Barbara* syllogism usually displays the linguistic form:

Every C is B
 Every B is A

 Every C is A

What has happened is that (i) premises and conclusion have been formulated in the 'new' way, and in addition (ii) the order of the premises has been turned round. As a result we have what we may dub the 'classical formulation' of categorical syllogisms. Apuleius' (or [Apuleius]) explication of these different formulations shows that this change was a conscious one:

... the first indemonstrable may also be <set out> in the manner of the Peripatetics, using letters, and with the order of premises and parts changed, but with the force remaining the same: A of every B, and B of every C, therefore A of every C. They begin from the predicate and also from the second premise. In fact, this mood—carried out backwards from their point of view—is thus: every C is B, every B is A, therefore every C is A.

(Apul. *Int.* 212.4–10 Moreschini)

*** ut etiam Peripateticorum more per litteras ordine propositionum et partium commutato sed vi manente sit primus indemonstrabilis: A de omni B, et B de omni C; igitur A de omni C. incipiunt a declarante atque ideo et a secunda propositione. hic adeo modus secundum hos pertextus retro talis est: omne C B, omne B A; omne igitur C A.

(*Peripateticorum* Prantl: *hypotheticorum* MSS)

We can now see how this change in formulating categorical propositions brought

¹³ Alexander provides a different reason why Theophrastus called them 'by analogy': their premises are similar (in form) to each other and to the conclusion (*An.Pr.* 326.10–12). But this need not have been Theophrastus' reason for the name. Cf. Barnes (n. 3), 288, n. 2.

¹⁴ For Alexander (who when commenting on Aristotle naturally mostly sticks to Aristotle's formulation), see e.g. *An.Pr.* 348.17.

about the reversal of the second and third hypothetical figures. In Aristotle's terminology, and using Theophrastus' ordering of the figures, we obtain the following analogy between the premise pairings of the categorical and hypothetical syllogisms:

First figure

A of **B** If (it is) A, (it is) **B**
B of C If (it is) **B**, (it is) C

Second figure

B of A If (it is) **B**, (it is) A
B of C If (it is) **B**, (it is) C

Third figure

A of **B** If (it is) A, (it is) **B**
C of **B** If (it is) C, (it is) **B**

(where 'X of Y' is short for 'X holds of Y', and quality and quantity are again disregarded). As in the previous case, the analogy is plain: the middle term (**B**) has the same position relative to the extremes (A, C) in the corresponding figures of the two types of syllogism. However, when categorical propositions were reformulated in the 'classical' way, the following mismatch would have appeared in the premise pairings of the second and third figures:

Second figure

C is **B** If (it is) **B**, (it is) A
A is **B** If (it is) **B**, (it is) C

Third figure

B is C If (it is) A, (it is) **B**
B is A If (it is) C, (it is) **B**

Here, in the second *categorical* figure, the common term now 'follows' the extremes (i.e. is predicated of them), whereas in the second *hypothetical* figure the common term still precedes; and similarly in the case of the third categorical and hypothetical figures. There is one easy way to restore the analogy, namely by having the second and third hypothetical figure swap places. And this, I take it, is precisely what happened. The reversal of the second and third hypothetical figures is thus a result (i) of understanding the hypothetical syllogisms as analogous to Aristotle's categorical ones, and (ii) of the change in the standard linguistic form of categorical propositions from Aristotle's own to the 'classical formulation'.

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